FORECASTING ENERGY INTENSITY WITH FOURIER RESIDUAL MODIFIED GREY MODEL: AN EMPIRICAL STUDY IN TAIWAN

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ABSTRACT: Energy intensity is defined as the energy consumption for producing every unit of real GDP in a certain time frame. Studies in forecasting the energy intensity have not well positioned due to the difficulty in collecting relevant data on the determinants affecting the energy consumption and GDP. Therefore, in this study, it is proposed to use the Grey forecasting model GM(1,1) to predict the energy consumption and real GDP before the intensity is forecasted. To enhance the accuracy level of the forecasting models, their residuals are then modified with Fourier series. In the case of Taiwan, the modified models resulted in very low values of mean of absolute percentage error (MAPE) of 0.33% and 0.58%, respectively to the energy consumption and real GDP. Hence, the modified model is strongly suggested to forecast the energy intensity in Taiwan from 2012-2015.

Keywords: GM(1,1), FGM(1,1), Grey forecasting, Fourier series, Energy intensity.

I. INTRODUCTION

Energy is the core of most economic, environmental and developmental issues around the globe. It has been well proved that there is a close relationship between the energy consumption and economic development. As per the definition offered by the Department of Economic and Social Affairs of the United Nations Secretariat, energy intensity is defined as the energy consumption for producing every unit of real GDP in a certain time frame which means that the lower the energy intensity of an economy is, the better the economy performs. Energy intensity indicates the total energy used to support a wide range of production and consumption (economic and social) activities [1]. Therefore, it is usually considered as one of the measures of sustainable development. A country with highly economical productivity, pleasant weather, geographically well-allocated work places, fuel efficient vehicles, mass transportation, etc., will have a far lower energy intensity and vice versa.

Many researches have been conducted and it has been found out that energy consumption and GDP are positively correlated though the correlation coefficients may be different from country to country [2]. Changes in the economy structure may result in using less additional energy; however, total energy consumption is still increasing [3]. In reducing the emissions through reducing the energy consumption, it was pointed out that the developed countries tend to be more affected by such policy rather than developing ones [4].

Many different researchers [5-15] have focused on the analysis of the relationship between the energy consumption and GDP. Nevertheless, the number of studies in forecasting the energy intensity is actually limited due to the fact that collecting relevant data on the determinants affecting the energy consumption and GDP runs into a lot of difficulties. Therefore, in this study, it is proposed to use the conventional Grey forecasting model GM(1,1), which has been widely used in different areas due to its ability to deal with the problems of uncertainty with few data points and/or “partial known, partial unknown” information, to predict the energy consumption and real GDP before the intensity is forecasted. To enhance the accuracy level of the forecasting model, its residuals are then modified with Fourier series. An empirical study in Taiwan is investigated as an example for this improved model.

II. LITERATURE REVIEWS

2.1 Grey Model

Grey theory offers a new approach to deal mainly with the problems of uncertainty with few data points and/or poor information which is said to be "partial known, partial unknown" [16]. The core of Grey theory is the Grey dynamic model which is usually called Grey model (GM). The Grey model is used to execute the short-term forecasting operation with no strict hypothesis for the distribution of the original data series [17]. The general GM model has the form of *GM(d,v)*, where d is the rank of differential equation and v is the number of variables appeared in the equation. The basic model of Grey model is *GM(1,1)*, a first-order differential model with one input variable. The procedure to obtain *GM(1,1)* is as the following:

Step 1: Suppose an original series with n entries is $x^{(0)}$:

$$x^{(0)} = \left\{ x^{(0)}(1),\ldots,x^{(0)}(k),\ldots x^{(0)}(n) \right\} \tag{1}$$

where $x^{(0)}(k)$ is the value at time k $\left(k = \overline{1,n} \right)$.

Step 2: From the original series $x^{(0)}$, a new series $x^{(1)}$ can be generated by one time accumulated generating operation (1-AGO), which is

$$x^{(1)} = \left\{ x^{(1)}(1),\ldots,x^{(1)}(k),\ldots,x^{(1)}(n) \right\} \tag{2}$$

where $$x^{(1)}(k) = \sum_{j=1}^{k} x^{(0)}(j)$$

Step 3: A first-order differential equation with one variable is expressed as:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b \tag{3}$$

where a is called a developing coefficient and b is called a grey input coefficient. These two coefficients can be determined by the least square method as shown below:

$$[a,b]^T = \left(B^T B \right)^{-1} B^T Y \tag{4}$$

where

$$B = \begin{bmatrix} -\left(x^{(1)}(1) + x^{(1)}(2) \right)/2 & 1 \\ -\left(x^{(1)}(2) + x^{(1)}(3) \right)/2 & 1 \\ \cdots & \cdots \\ -\left(x^{(1)}(n-1) + x^{(1)}(n) \right)/2 & 1 \end{bmatrix}$$

$$Y = \left[x^{(0)}(2), x^{(0)}(3),\ldots,x^{(0)}(n) \right]^T$$

Therefore, the solution of equation (3) is expressed as:

$$x^{(1)}(t) = \left[x^{(1)}(1) - \frac{b}{a} \right] e^{-at} + \frac{b}{a} \tag{5}$$

Equation (5) is also known as time response function of the equation (3). From equation (5), the time response function of the GM(1,1) is given by:

$$\hat{x}^{(1)}(k) = \left[x^{(0)}(1) - \frac{b}{a} \right] e^{-a(k-1)} + \frac{b}{a} \quad \left(k = \overline{1,n} \right) \tag{6}$$

Based on the operation of one time inverse accumulated generating operation (1-IAGO), the predicted series $\hat{x}^{(0)}$ can be obtained as the following:

$$\hat{x}^{(0)} = \left\{ \hat{x}^{(0)}(1),\ldots,\hat{x}^{(0)}(k),\ldots,\hat{x}^{(0)}(n) \right\} \tag{7}$$

where

$$\begin{cases} \hat{x}^{(0)}(1) = \hat{x}^{(1)}(1) \\ \hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1) & \left(k = \overline{2,n} \right) \end{cases}$$

2.2 Fourier Residual Modification

In order to improve the accuracy of forecasting models, the Fourier series has been widely and successfully applied in modifying the residuals in Grey forecasting model *GM(1,1)* which reduces the values of RMSE, MAE, MAPE, etc., [18-22]. The overall procedure to obtain the modified model is as the followings:

Let x is the orginal series of n entries and $\hat{x}$ is the predicted series obtained from *GM(1,1)*. Based on the predicted series $\hat{x}$, a residual series named ε is defined as:

$$\varepsilon = \{ \varepsilon(2), \varepsilon(3), \varepsilon(4),\ldots,\varepsilon(k),\ldots,\varepsilon(n) \} \tag{8}$$

where

$$\varepsilon(k) = x(k) - \hat{x}(k) \qquad \left(k = \overline{2,n} \right)$$

Expressed in Fourier series, $\varepsilon(k)$ is rewritten as:

$$\varepsilon(k) = \frac{1}{2} a_0 + \sum_{i=1}^{F} [a_i \cos(D) + b_i \sin(D)] \tag{9}$$

where $D = 2\pi ik/(n-1)$ $\qquad \left(k = \overline{2,n} \right)$

where $F = [(n-1)/2-1]$ called the minimum deployment frequency of Fourier series [21] and only take integer number [18-20].

And therefore, the residual series is rewritten as:

$$\varepsilon = P.C \qquad (10)$$

where

$$P = \begin{bmatrix} \frac{1}{2} & \cos\left(\frac{2\pi\times 1}{n-1}\times 2\right) & \sin\left(\frac{2\pi\times 1}{n-1}\times 2\right) & \cdots & \cos\left(\frac{2\pi\times F}{n-1}\times 2\right) & \sin\left(\frac{2\pi\times F}{n-1}\times 2\right) \\ \frac{1}{2} & \cos\left(\frac{2\pi\times 1}{n-1}\times 3\right) & \sin\left(\frac{2\pi\times 1}{n-1}\times 3\right) & \cdots & \cos\left(\frac{2\pi\times F}{n-1}\times 3\right) & \sin\left(\frac{2\pi\times F}{n-1}\times 3\right) \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{1}{2} & \cos\left(\frac{2\pi\times 1}{n-1}\times n\right) & \sin\left(\frac{2\pi\times 1}{n-1}\times n\right) & \cdots & \cos\left(\frac{2\pi\times F}{n-1}\times n\right) & \sin\left(\frac{2\pi\times F}{n-1}\times n\right) \end{bmatrix}$$

$$C = [a_0, a_1, b_1, a_2, b_2, \ldots, a_F, b_F]^T$$

The parameters $a_0, a_1, b_1, a_2, b_2, \ldots, a_F, b_F$ are obtained by using the ordinary least squares method (OLS) which results in the equation of:

$$C = \left(P^T P\right)^{-1} P^T [\varepsilon]^T$$

Once the parameters are calculated, the modified residual series $\hat{\varepsilon}$ is then achieved based on the following expression:

$$\hat{\varepsilon}(k) = \frac{1}{2}a_0 + \sum_{i=1}^{F}[a_i \cos(D) + b_i \sin(D)] \qquad (11)$$

From the predicted series $\hat{x}$ and $\hat{\varepsilon}$, the Fourier modified series $\breve{x}$ of series $\hat{x}$ is determined by:

$$\breve{x} = \{\breve{x}(1), \breve{x}(2), \ldots, \breve{x}(k), \ldots, \breve{x}(n)\} \qquad (12)$$

where

$$\begin{cases} \breve{x}(1) = \hat{x}(1) \\ \breve{x}(k) = \hat{x}(k) + \hat{\varepsilon}(k) & \left(k = \overline{2,n}\right) \end{cases}$$

To evaluate the model accuracy, there are four important indexes to be considered, such as:

- The mean absolute percentage error (MAPE) [19, 22, 23]:

$$MAPE = \frac{1}{n}\sum_{k=1}^{n}\frac{|x(k)-v(k)|}{x(k)} \qquad \left(k = \overline{1,n}\right)$$

where *v(k)* is the forecasted value of *k*th entry from the model ($v(k) = \hat{x}(k)$ in GM(1,1) or $v(k) = \breve{x}(k)$ in FGM(1,1)).

- The post-error ratio *C* [24, 25]:

$$C = \frac{S_2}{S_1}$$

where

$$S_1 = \sqrt{\frac{1}{n}\sum_{k=1}^{n}[x(k)-\bar{x}]^2} \qquad \bar{x} = \frac{1}{n}\sum_{k=1}^{n}x(k)$$

$$S_2 = \sqrt{\frac{1}{n}\sum_{k=1}^{n}[\varepsilon(k)-\bar{\varepsilon}]^2}$$

$$\varepsilon(k) = x(k) - v(k) \qquad \bar{\varepsilon} = \frac{1}{n}\sum_{k=1}^{n}\varepsilon(k)$$

The smaller the *C* value is, the higher accuracy the model has.

- The small error probability *P* [24, 25]:

$$P = p\left\{\frac{|\varepsilon(k)-\bar{\varepsilon}|}{S_1} < 0.6745\right\}$$

The higher the *P* value is, the higher accuracy the model has.

- The forecasting accuracy ρ [25]:

$$\rho = 1 - MAPE$$

The above four indexes are used to classify the grades of forecasting accuracy as in Table 1.

Table 1. Four grades of forecasting accuracy

Grade level	***MAPE***	***C***	***P***	$\boldsymbol{\rho}$
I (Excellent)	< 0.01	< 0.35	> 0.95	> 0.95
II (Good)	< 0.05	< 0.50	> 0.80	> 0.90
III (Qualified)	< 0.10	< 0.65	> 0.70	> 0.85
IV (Unqualified)	≥ 0.10	≥ 0.65	≤ 0.70	≤ 0.85

III. EMPIRICAL RESULTS

The data of energy consumption from 1999 – 2011 in Taiwan are obtained from the Bureau of Energy of Ministry of Economic Affairs of Taiwan [26]; whereas the data of the Taiwan GDP from the same period are collected from International Monetary Fund [27]. Only data from 1999-2010 are used to build relevant *GM(1,1)* and *FGM(1,1)* models. The data in 2011 is used to compare with the forecasted value from the selected model to further affirm its forecasting power.

3.1. Forecasting model for the energy consumption

Based on the algorithm expressed in section 2.1, the fundamental Grey forecasting model for the energy consumption named $GM(1,1)_E$ is found as the following:

$$\hat{x}^{(1)}(k) = 3377369.76e^{0.030273(k-1)} - 3285397.26$$

The residual series attained from $GM(1,1)_E$ is then modified with Fourier series, which results in the modified model $FGM(1,1)_E$ as per the algorithm stated in section 2.2. The evaluation indexes of $GM(1,1)_E$ and $FGM(1,1)_E$ are summarized as in Table 2. Table 2 clearly showed that between $GM(1,1)_E$ and $FGM(1,1)_E$, $FGM(1,1)_E$ is selected because it has a lower value of MAPE and a better forecasting power. So, $FGM(1,1)_E$ is used to forecast the energy consumption in 2011. The forecasted value is then compared with the actual consumption in order to further affirm its forecasting power as shown in Table 3. The MAPE value of 5.55% indicates that $FGM(1,1)_E$ can be appropriately used to forecast the consumption in 2012 – 2015. The forecasted values in this period are shown in Table 4.

3.2. Forecasting model for the GDP

Similarly, the fundamental Grey forecasting model for the GDP named $GM(1,1)_G$ is found as the following:

$$\hat{x}^{(1)}(k) = 8943.01e^{0.032091(k-1)} - 8667.89$$

$FGM(1,1)_G$ is accordingly obtained based on section 2.2. It is also selected because it outperforms $GM(1,1)_G$ in term of low MAPE value as shown in Table 2. Its forecasted value of GDP in 2011 shown in Table 3 has an MAPE value of 1.83% indicating that it can be used to forecast the GDP in 2012 – 2015. Its relevant forecasted values are also shown in Table 4.

Table 2. Summary of evaluation indexes of model accuracy

Model \ Index	MAPE	S1	S2	C	P	ρ	Forecasting power
$GM(1,1)_E$	0.0352	14781.86	5032.84	0.34	1.00	0.9648	*Good*
$FGM(1,1)_E$	0.0033	14781.86	475.76	0.03	1.00	0.9967	*Excellent*
$GM(1,1)_G$	0.0414	42.22	15.94	0.38	1.00	0.9586	*Good*
$FGM(1,1)_G$	0.0058	42.22	2.31	0.05	1.00	0.9942	*Excellent*

Table 3. Forecasted energy consumption and GDP in 2011

Model	Unit	Actual value	Forecasted value	MAPE
$FGM(1,1)_E$	10^3 KLOE	131,832.50	139148.40	0.0555
$FGM(1,1)_G$	10^9 USD	430.58	422.71	0.0183

Table 4. Forecasted energy consumption and GDP from 2012-2015

Item	Unit	2012	2013	2014	2015
Energy	10^3 KLOE	148519.70	147592.90	152744.00	157530.00
GDP	10^9 USD	447.45	454.54	465.26	479.98
Energy intensity	KLOE/10^6USD	331.92	324.71	328.30	328.20

Table 4 shows that there is a small decrease in the energy intensity index of Taiwan in the coming years. This could be explained as an outcome of the past and current investment in the new production technology as well as the modern facilities in the transportation, services and residential sectors. Besides, the moving of its manufacturing factories to other countries including China, Vietnam, Indonesia, Malaysia, Laos, Thailand, etc., as well as the enhancing of its service industries make Taiwan not only consume less energy but also produce higher GDP, which significantly contribute to the decrease of the energy intensity index of Taiwan.

IV. CONCLUSION

The accuracy level of the traditional Grey forecasting model *GM(1,1)* can be well improved if the model is modified with Fourier series. In the case of energy intensity of Taiwan, with the Fourier modified Grey forecasting model *FGM(1,1)*, it was found out that the energy intensity of Taiwan becomes lower and lower representing a better & stable development of the country. This result plays as an excellent motivation for the authorities to assert that they are on the right way to develop Taiwan in general and its economy in particular. Other countries could refer to this as a good example to focus on research & development as well as invest and apply advanced technology in most of their activities.

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